~~THE~~ INTERSEGMENTAL COMMITTEE OF THE ACADEMIC SENATES

Statement on Competencies in Mathematics Expected of Entering College Students

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1 INTRODUCTION

The goal of this Statement on Competencies in Mathematics Expected of Entering College Students, or Mathematics Competencies Statement, is to provide a clear and coherent message about the mathematics that students should understand and be able to perform in order to be successful in college. Learning appropriate mathematics content during high school is critical to student success in higher education. This Mathematics Competencies Statement is intended to assist high school teachers, counselors, and administrators, as well as students and their families, in the selection of secondary mathematics pathways that best prepare students for higher educational opportunities. The Mathematics Competencies Statement should also be useful to anyone who is concerned about the preparation of California's students for college. This document represents an effort to be realistic about the skills, approaches, experiences, and subject matter that make up an appropriate mathematical background for entering college students.

High school students are rapidly maturing as young people and increasingly question the relevance of everything they encounter. Thus, mathematics must be presented as an engaging subject that empowers students to explore their interests, career aspirations, and potential college pathways. These interests and career goals often change as a student moves through the last years of high school and the first years of college. Consequently, incoming college students should learn more than the mathematics required for a single program or major. Instead, the essential curriculum for college bound students should empower them to access the full range of knowledge afforded by a college education as well as to navigate their mathematical options in alignment with their interests, majors, and professional goals.

A college education provides breadth as well as the opportunity to specialize in a particular discipline. All three public higher education systems in California—the University of California (UC), the California State Universities (CSU), and the California Community Colleges—expect students to engage in science, technology, and mathematics as part of their general education requirements. California Code of Regulations Title 5 §55061 puts it well:

> General Education is designed to introduce students to the variety of means through which people comprehend the modern world. It reflects the conviction of colleges that those who receive their degrees must possess in common certain basic principles, concepts and methodologies both unique to and shared by the various disciplines. College educated persons must be able to use this knowledge when evaluating and appreciating the physical environment, the culture, and the society in which they live.

Each of the three systems of higher education has designed its own requirements and expectations:

- All graduating UC students must complete general education requirements—sometimes called breadth requirements—that are designed to give undergraduates a broad base of knowledge. Although individual colleges at each UC campus have varied general education requirements, they typically include at least one course in mathematics or quantitative reasoning and an additional requirement in a quantitative field such as physical or biological science.

- All graduating CSU students "shall have completed a program which includes… [a] minimum of 13 semester (or quarter equivalent) units… to include inquiry into the physical universe and its life forms, which includes a 1 semester (or quarter equivalent) unit laboratory activity, and mathematical concepts and quantitative reasoning and their applications." [Title 5 §40405.1]
- Individual community college districts have their own general education expectations, but students who transfer to the UC or CSU must satisfy the requirements of the university they will attend.

Therefore, entering college students need to be ready to meet the expectations of an educational system that exposes them to a wide variety of knowledge.

The intensity of most majors regarding quantitative reasoning and applications is steadily increasing as fields of study take advantage of the greater availability of data. A quick look at Advanced Placement exams in history, psychology, biology, economics, and statistics shows the growing importance for practitioners to reason quantitatively by fitting data to basic mathematical function models, interpolating for missing information, or extrapolating information to make predictions. In addition to the increasing use of data, more scientific approaches to societal problems also demand mathematical competence. Many majors leading to high-demand jobs—such as nursing, physical therapy, marketing, engineering management, and computer information technology—may not require the full power of calculus, but they do require chemistry, physics, computer science, or other quantitatively intense coursework.

The growth of quantitative reasoning in all subject areas, as well as the general education requirements of all three higher education systems, reinforces the message that the time to build up a solid core of flexible mathematical skills is in the first three years of high school. The last year of high school and first year of college can then be leveraged to explore different areas of interest. The base provided in the first three years, together with a fourth year of mathematical exploration, is intended to ensure that students have equitable educational opportunities that allow them to thrive in an academic pathway chosen from a broad range of options rather than being limited by choices made in high school. If students want to maximize their access to a wide choice of college majors, then they are well-served to enter college with a strong grounding in the mathematical competencies expected of entering college students.

Section 2 of this Statement on Mathematics Competencies provides some background information on the California higher educational systems, the history of this document, and other materials that are relevant to mathematics education in California. Section 3 describes qualitative aspects of mathematical understanding that are critical to students' confidence and resilience in mathematics. Section 4 delves into the specific topics that all incoming college students are expected to learn in their first three years of high school. Section 5 looks at how mathematical proficiency can be advanced in the senior year to support students within varied disciplines. While disciplines are commonly divided into STEM and non-STEM—or quantitative and non-quantitative—section 5 takes a more fine-grained approach by considering six different meta-majors. These collections of college majors have common core courses, related content, and similar mathematics requirements.

2 BACKGROUND AND CONTEXT

This document articulates mathematical competency expectations for students matriculating to any of the three segments of California's public higher education system: the University of California, the California State University, and the California Community Colleges. It is published by the Intersegmental Committee of the Academic Senates (ICAS), which consists of representatives from the separate Academic Senates of the three segments. ICAS regularly discusses issues of mutual concern, including preparation for postsecondary education, transfer, and course articulation. In this capacity, ICAS convened a subcommittee of faculty from each higher educational segment to author this document in support of stronger and more equitable student outcomes in mathematics and quantitative reasoning and to help students best meet mathematical expectations at the postsecondary level. This document updates the 2013 Statement on Mathematics Competencies Expected of Entering College Students to reflect changes to the California curricular landscape.

The ICAS subcommittee that prepared this document was composed of subject matter experts in statistics, data science, pure mathematics, and applied mathematics. ICAS instructed the subcommittee to support stronger and more equitable student outcomes in mathematics and quantitative reasoning. The subcommittee members were committed to identifying mathematical competencies that provide all students with the opportunity to choose among a broad spectrum of college majors. To that end, the subcommittee circulated a questionnaire to over two thousand faculty in the three California higher education segments. The questionnaire asked respondents about the importance of specific mathematical competencies for students entering each respondent's degree program. Three open-ended questions gave respondents the opportunity to identify skills that they considered particularly important or that had not been mentioned in the questionnaire and to comment on students' mathematical preparedness for their fields.

The committee received over 1,800 completed questionnaires. Importantly, this feedback came from higher education instructors from a diverse set of disciplines, including those outside of mathematics, science, and engineering programs. The high level of response and the evident effort in replying to the open-ended questions showed that faculty considered mathematical preparation to be important for student success. After the subcommittee authored the first draft of this document, it was shared with key stakeholders to provide additional feedback before a final version was presented to ICAS for review and approval.

The 2013 Mathematics Competencies Statement was itself an update to two previous versions—2010 and 1997—to take account of the California Common Core State Standards in Mathematics, which were adopted in 2010 and revised in 2013. The bulk of the 2013 statement was unchanged from the 1997 statement. The 2013 statement, like the previous versions, was divided into two sections. The first section described the characteristics and dispositions of a student who is properly prepared for college and aspects of instruction to foster those characteristics. The second section of the 2013 statement had specific subject matter recommendations that were organized into a base level, "essential," and an aspirational level, "desirable," for each of two categories of majors, quantitative and non-quantitative.

The current version of the Mathematics Competencies Statement is restructured, although it borrows heavily from the previous document. Section 3, which describes overarching qualitative aspects of mathematical understanding, integrates some of the earlier material on the characteristics and dispositions of a well-prepared student. Section 4 presents the core material that every student should know after successfully completing the standard first three years of high school coursework—e.g., Algebra 1, Geometry, Algebra 2, or Integrated Mathematics 1, 2, 3—and the material contained in the K-8 curriculum. The topics are defined in more detail than in the 2013 statement, with descriptions of the skills that students should acquire. Section 5 focuses on the fourth year of mathematics and uses a finer breakdown of possible majors than quantitative and non-quantitative. It presents recommendations for six meta-majors, or groupings of related majors, and provides suggestions for fourth year courses.

Updating the statement in 2024 is particularly important because preparatory mathematics courses in the three California higher education segments have changed considerably since the 2013 version of the document.

- After the passage of AB 705 (Irwin, 2017), California community colleges shifted their approach away from offering developmental mathematics courses. Mathematics remediation has been replaced with co-requisite support classes, stretch classes, or other similar approaches. With the passage of AB 1705 (Irwin, 2022), community colleges are required to place and enroll students, with a few exceptions, into a math class that meets the requirement of their major. The subsequent AB 1705 Implementation Guide (Stanskas, 2024) authored by the California Community Colleges Chancellor's Office guides the colleges to place students intending to major in programs that require calculus either directly into calculus, or, in specified circumstances, into calculus preparatory classes. Thus, the transfer-level preparatory courses leading up to calculus may be less available to community college students than they were before AB1705.
- After CSU Executive Order 1110 (Forbes, 2017), the CSU campuses also shifted their approach away from offering developmental mathematics courses to co-requisite support classes and other similar approaches.
- For the UC system, courses that prepare students for calculus, statistics, and other college-level mathematics courses vary by institution. On some UC campuses, classes below the level of pre-calculus are offered, but UC Academic Senate Regulation 761.A requires that these courses be counted as workload credit and not for baccalaureate credit (University of California Academic Senate, n.d.).

In general, high school graduates attending a California public higher educational institution are expected to start directly in the college credit-bearing mathematics classes required for their major or that support their transfer plans. Thus, choosing appropriate mathematics coursework at the high school level has an even more significant impact on a student's academic trajectory in college than it did in 2013.

This Mathematics Competencies Statement presents what students should know when they enter college, but the relationship between this document and the primary document guiding policy around K-12 mathematics education in California, the 2013 California Common Core State Standards in Mathematics (CA CCSSM) is also important to note. The CA CCSSM provides a detailed description of content to be delivered at each grade level and presents a strategy for students› growth in knowledge, skill, and sophistication in mathematical subjects as they proceed from one grade to another. These standards were designed to develop in students the «knowledge and skills that young people will need for success in college and careers» (California State Board of Education, 2013). The standards emphasize real world relevance in addition to procedural skill and fluency using three key principles: focus on the concepts listed in the standards so that students have a strong foundation in key material, coherence of the various areas of mathematics by showing interrelationships among them, and rigor, or the requirement that conceptual understanding, procedural skill, and applications are equally important. Throughout this Mathematics Competencies Statement, the values of the CA CCSSM—real world relevance, coherence of mathematics, and rigor—should be evident.

The CA CCSSM has two types of standards. First, the Standards for Mathematical Content detail topics to be covered at each grade level. The topics listed in Section 4 of this document are all found in the CA CCSSM Standards. Many of them may be introduced at one grade level and then developed further in later grades. The discussion in this document provides focus on the mathematical topics considered most critical for success in college and the understanding of these topics that students should have developed upon completion of their high school education. Second, the CA CCSSM Standards for Mathematical Practice are independent of grade level and address habits of mind that students should develop. Section 3 of this document, Qualitative Aspects of Mathematical Competency, treats broad themes—conceptual understanding, reasoning, and problem solving—and also addresses how students should approach mathematics.

This Mathematics Competencies Statement is appearing at a time of renewed concern about mathematics education in California and deepened attention to disparities in educational outcomes and inequitable access to educational resources. The 2023 California Mathematics Framework, which updates the 2013 California Mathematics Framework, has a strong focus on equity. Chapter 1, Mathematics for All, identifies three aspects of equitable instruction that should be understood by teachers and students (California Department of Education, 2024):

- "everyone is capable of learning math and that each person's math capacity grows with engagement and perseverance";
- Diversity should be treated as an asset; and
- Instruction should include a multidimensional approach using visualization, models, and common language along with computations, formulas, and graphs.

The 2016 CSU Quantitative Reasoning Task Force report (QRTF) has as its guiding principle to balance two aspects of equity. One is equitable access to CSU admission and the opportunity to graduate with a university degree. The other is that a university degree should provide students with excellent training that allows graduates to compete for rewarding and lucrative career opportunities. The task force recognized that imposing mathematics entrance requirements risks excluding students from the university. On the other hand, with lower entrance requirements, a college education is built on a narrower foundation, which risks limiting the value of the degree (Academic Senate of the California State University, 2016).

This 2024 Mathematics Competencies Statement shares the perspective on equity expressed in the QRTF report. The objective of this document is to provide the perspective of college faculty on material that is critical for success in college as well as how students should understand and work with that material. By making the expectations clear in a concise and comprehensible statement, this document contributes to an equitable environment for mathematics education. The long-term goal is that all students will arrive at college well-prepared to engage with challenging material, advance in their chosen discipline, and achieve a college degree that provides entry to rewarding careers and meaningful contributions to society.

3 QUALITATIVE ASPECTS OF MATHEMATICAL COMPETENCY

Certain overarching aspects of mathematical competency provide essential framing for the remainder of this document. While later sections will indicate the specific topics expected for students entering college, this section highlights essential qualitative aspects of learning mathematics that help organize students' mathematical knowledge into a coherent web. This broad perspective reinforces the relationships between diverse mathematical topics and is intended to prevent the later content from being interpreted as a disconnected checklist.

a. Conceptual Understanding, Abstraction, and Generalization

Internalizing basic mathematical skills should not be mistaken as the main objective of a secondary mathematics education. As students learn mathematics, they must also look for patterns and relationships that provide a framework to connect mathematical topics to one another. Mathematics is not a collection of disjointed facts; rather, it is composed of principles that glue diverse topics together. For example, one recurrent theme in mathematics is that different expressions can have the same value, and having a standard form for that value—e.g., writing a fraction in its simplest or reduced form—is often useful. Another key theme is the properties of equality, which should become familiar and reasonable to students as they encounter increasingly advanced material in which they solve equations. At a more advanced level, functions are used to characterize the relationship between two quantities, one dependent on another. This abstract concept is used in tremendously varied real-world applications: population studies, economics, computer programming, and climate analysis to name just a few. As students repeatedly see fundamental concepts in different contexts, they can begin to appreciate how mathematics provides a general framework that can be applied in many different settings.

b. Analytic Reasoning and Communication

Analytic reasoning is the construction of a compelling argument based on a sequence of logical statements to explain why a particular claim is true. Levels of analytic reasoning include explaining the steps in a computation, using a definition, providing concrete examples, giving informal arguments using words and pictures, and, at the advanced level, presenting a formal proof. Students entering college should be comfortable questioning and exploring why one statement follows logically from another. They should expect to have their understandings challenged with questions that cause them to further examine, refine, and explain their reasoning. They should be prepared for classrooms that are full of discourse and interaction focused on reasoning and logic. Ideally, students are driven by curiosity to understand the justifications for statements they are taught, and as they learn to analyze and reason well, they become more independent and resilient learners.

c. Problem Solving and Mathematical Modeling

Problem solving is the essence of mathematics. It is not a collection of specific techniques to be learned, and it cannot be reduced to a set of procedures. It is the experience of being confronted with an unfamiliar problem, breaking the problem into approachable parts, developing a planned approach, implementing and revising that plan iteratively, and ultimately discussing various attempts at solutions. Students entering college should have had successful experiences solving problems and reflecting on the problem-solving process. Problems can be solved in a variety of different ways, and analysis of alternative methods not only enables one to evaluate their advantages but is also a key to propelling mathematical understanding. Experience in solving problems gives students the confidence and skills to approach new situations creatively by modifying, adapting, and combining mathematical tools.

An integral part of problem solving in the real world is mathematical modeling, the process by which mathematical concepts and techniques are used to represent, analyze, predict, or otherwise provide insight into real-world phenomena. A well-prepared college student will be able to translate from the contextualized information provided in the problem's formulation to mathematical information in the form of numbers, variables, equations, inequalities, and other mathematical objects. Such a student will additionally be able to use mathematical methods to arrive at a solution and interpret the results in the original context.

Students need experience working with models of many forms, including deterministic models, which give a specific set of possible solutions, and inferential models, where the likelihood of solutions is considered. The ability to assess what type of information is provided in the problem and to formulate a suitable mathematical model is developed over many years, starting with very simple word problems that students encounter in elementary school. A well-prepared student will have been continuously and consistently exposed to problem solving and modeling throughout K-12. Such exposure will prepare students to master even more sophisticated mathematical tools in college and use them effectively to address problems from a variety of applications.

d. Dispositions Toward Mathematics

The goal of developing these higher-level learning experiences—conceptual understanding, logical reasoning, and problem solving—is to foster in students the following dispositions toward mathematics and, as a consequence, to help them enjoy the material more, learn and retain it more easily, and see its utility more deeply.

- Mathematics makes sense: Students should perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized and applied.
- Mathematics is a unified field of study: Students should see interconnections among various areas of mathematics taught in different courses that are often perceived as distinct.
- Assertions require justification based on logically sound arguments: Students should habitually ask, “Why?” They should expect convincing answers to their questions, and they should be able to provide convincing explanations for their work.
- Mathematical problems require time and thought: Students should be willing to experiment, make mistakes, and be tenacious; they should not expect to simply mimic examples that have already been seen.
- Clear and coherent communication is important: Students should aspire to speak and write about mathematical topics using both formal and natural language to communicate effectively with peers and teachers.
- Learning is a collaborative effort: Students should realize that their minds are their most important mathematical resources and that teachers and other students can help them to learn but cannot learn for them.

4 CORE COLLEGE-PREPARATORY COMPETENCIES FROM THE INITIAL YEARS OF HIGH SCHOOL MATHEMATICS

This section presents the mathematical skills that provide a foundation for academic success in college. The information is divided into topics that roughly follow those of the California public educational system, and the structure endeavors to show how topics covered early in an academic journey—arithmetic, basic geometry, and algebra—form a foundation for more advanced topics like statistics and the study of functions. Specific skills that students should acquire in high school are listed, yet they must be seen as interconnected concepts rather than as discrete skills isolated from one another. Without an emphasis on connections, mathematical learning becomes a fragile tower of unrelated facts. Understanding the connections among the topics in geometry, algebra, statistics, and applications of these subjects is the foundation of what college-bound students need to succeed. This broad view allows for a more stable and flexible knowledge base, allowing students to utilize mathematics to solve problems and make sense of the world around them.

Many of the topics listed below are introduced in elementary or middle school. They appear here to emphasize their importance as a foundation for more advanced topics. Students are not expected to have the same level of fluency with everything that is listed here, and college instructors know that more advanced topics, such as the use of functions and complex algebraic manipulations, will require some review. However, students will be expected to have a high level of competence and confidence in foundational material and the ability to rapidly recall and employ the more advanced material after review. Students will also have seen a variety of ways to represent mathematical concepts: using everyday language, algebraic expressions, and graphical representations as well as diagrams or pictures. They may have used spreadsheets and other software platforms to do simulations or computations. The ability to work with a variety of representations and tools continues to be important in many college courses, not just those in mathematics but also in sciences, social sciences, and other fields.

a. Number Sense, Measurement, and Arithmetic

An understanding of number systems is a foundation for all mathematics. A student should be competent in the use of ratios, estimation, numerical precision, order of magnitude, and units of measurement. The meanings of arithmetic operations and physical or geometric interpretations of the operations are particularly important. A student who is college ready will be able to interpret real-world quantitative problems, translate them into arithmetic operations, perform the operations, and explain the solution in the context of the original problem.

The following are typical expectations for college-bound students:

- Translate between ratios, fractions, decimals, and percentages.
- Convert from one unit of measure to another and use a rate of change appropriately; e.g., find distance from speed and time or compute density from weight and volume.
- Use the equal sign appropriately and understand that it indicates that two different arithmetic expressions have the same value.
- Simplify arithmetic expressions involving sums, products, and exponents using appropriate order of operations and the properties of commutativity, associativity, and distributivity.
- Understand and use integer factorization in computations involving fractions.
- Understand the properties of exponents and use exponents correctly, including negative and rational exponents.
- Work with scientific notation and understand order of magnitude and approximation.
- Estimate the outcome of an arithmetic computation and understand the utility of estimation in practical problems.
- Plot points on a number line—including decimals, fractions, and square roots—and interpret relationships among points plotted on a number line.

Emphasis should be placed on developing a sense for numbers, numerical precision, order of magnitude, and the interpretation of arithmetic operations. Facility with computations of small numbers is valuable, while complicated numerical computations can be done by computer and checked for reasonableness such as correct order of magnitude.

b. Variables, Equations, and Algebraic Expressions

A major advance in mathematical sophistication comes in algebra, in which a variable is used to represent a quantity of which the value is unspecified. Much of the computational manipulation in algebra is based on the properties of arithmetic established in elementary school; the challenge is to do the computations abstractly, with letters representing unknown values. Students should appreciate that plotting a graph of an algebraic equation in two or three variables is a useful process that reveals information and aids visualization and intuition about algebraic relationships. Plots can be used to explain how the value of an algebraic formula varies with different values of one of the variables. A student who is college ready will be able to interpret real-world quantitative problems involving varying or unknown quantities, translate them into algebraic expressions, and then solve, simplify, or substitute values to obtain information and explain the solution in the context of the original problem.

The following are typical expectations for college-bound students:

- Evaluate, simplify, and manipulate algebraic expressions while using the equal sign appropriately to show that different algebraic expressions are equal.
- Solve linear equations and inequalities in one variable—including those using absolute value—and represent the solutions on a number line.
- Use the equal sign appropriately in the process of solving an equation by deriving simpler but equivalent equations.
- Graph an equation for a line, interpret slope as a rate of change, and derive the equation for a line from its graph.
- Solve two linear equations in two variables and relate the solution to graphical representations.
- Graph solutions of linear inequalities in two variables.
- Graph a circle from an equation, derive the equation of a circle from a graph, and interpret circles as points that are equidistant from a center.
- Manipulate quadratic expressions and solve quadratic equations in one variable by factoring or completing the square.
- Manipulate rational expressions of polynomials where the denominator has degree one and solve equations with these rational expressions.
- Manipulate exponential expressions of the form ra^x and solve simple exponential equations using logarithms.
- Translate from a quantitative problem written in English to an algebraic expression, equation, or inequality as appropriate and relate solutions derived algebraically back to the original problem.

Emphasis should be placed on algebra as a language for describing mathematical relationships and as a means for solving problems. Algebra should not merely be the implementation of a set of rules for manipulating symbols. The ability to use graphical representations to aid visualization and intuition about algebraic relationships is crucial.

c. Geometry and Trigonometry

Students are introduced to geometric concepts early in their educational journeys when they identify different shapes and compute measurement quantities such as the perimeter, area, or volume of a figure. As students progress, they explore conjecture and proof at a very basic level by reasoning from Euclid's axioms to derive statements about congruence of figures, parallelism, or other properties. As they enter college, students should be able to reason geometrically and provide compelling arguments for results. For example, a student can demonstrate insight into geometry by decomposing a complex shape into simpler ones in order to compute its area or by arguing for the congruence of figures based on fundamental principles.

The following are typical expectations for college-bound students:

- Identify two-dimensional and three-dimensional shapes and objects such as squares, rectangles, triangles, circles, rectangular solids, spheres, cones, and pyramids and compute the area, perimeter, surface area, and volume, as appropriate, of these objects.
- Make arguments about the congruence or similarity of figures using congruent sides, congruent angles, or other measurement information.
- Apply geometric transformations—rotation, reflection, translation, dilation—to a figure, sketch the resulting figure, and comment on the relationship between the original and transformed figures.
- Identify symmetries of a geometric figure.
- Apply the Pythagorean theorem.
- Determine the distance between two points in the coordinate plane.
- Use definitions of trigonometric ratios to determine relationships between angle measurements and lengths of the sides of a right triangle.
- Apply trigonometric ratios to find coordinates of points on the unit circle.

Emphasis should be placed on developing an understanding of geometric concepts sufficient to solve unfamiliar problems. The ability to provide compelling geometric arguments from established first principles is much more important than memorization of terminology and formulas.

d. Data, Statistics, and Probability

In the twenty-first century, data has become ubiquitous. It is used on a regular basis in politics for elections, polls, and campaigns as well as in advertising, medicine, education, sports, criminology, insurance, finance, and many more areas. The prevalence of machine learning and artificial intelligence tools that use algorithms to recognize patterns and make predictions underscores the importance of understanding the various methods of data collection and analysis and how the methods used can impact outcomes.

The following are typical expectations for college-bound students:

- Use basic descriptive statistics to represent key features of data, such as numerical or graphical representations of measures of center, variability, or shape.
- Formulate statistical investigative questions and either justify why a question can be answered based on the available data or specify any additional data needed.
- Recognize the differences between qualitative and quantitative variables.
- Analyze distributions of numerical data by using summary statistics, such as by finding and interpreting the mean, median, standard deviation, and interquartile range for a small sample of numeric data.
- Apply the basic definitions of probability to find the probability of an event in standard examples of a sample space, such as a coin toss, roll of dice, or drawing from a deck of cards.
- Create and interpret graphs and other data visualizations with and without technology, such as a bar graph, box plot, or scatter plot, as a way to describe distributions of data and the relations between variables.
- Explain and critique statistical statements, charts, and graphs found in news, reports, and advertisements and evaluate the strengths and limitations of conclusions drawn from incomplete information.
- Identify the uses and misuses of data that can take place in data analysis.

Emphasis should be placed on making sense of data and critically analyzing information derived from the data. Students should see statistical reasoning as a problem-solving process involving formulating an investigative question, determining which data is to be collected and which measures are used, presenting results with graphs and summary statistics, and interpreting results. As they participate in statistical investigations, students should become aware that the world around them is filled with data that can affect their lives, and they should begin to appreciate that statistics can help them make decisions based on data.

e. Analytical Reasoning and Justification

Students should perceive mathematics as a subject that inherently makes sense. They should understand that mathematical derivations, graphing, and other activities provide order and meaning rather than viewing mathematical concepts as a collection of arbitrary techniques to memorize. The terminology, computations, algorithms, and methods utilized in mathematics are often rooted in observations of the natural world. They have been formalized into precise rules that enable the prediction and discovery of new concepts and phenomena that advance human understanding of the world. Students should appreciate that by applying these rules one can derive conclusions that would otherwise be inaccessible and that one can identify similar structures across radically different situations. A robust mathematics education should also emphasize the importance of justifying assertions through persuasive and logical arguments. Students can expect this practice from their teachers but should also be equipped to articulate their own understanding of problems to both peers and educators.

The following are typical expectations for college-bound students:

- Justify the steps in the solution of an equation or simplification of an expression by referring to fundamental principles of algebraic operations—e.g., the distributive law—and of equality, e.g., adding the same quantity to both sides of an equation yields an equivalent equation.
- Explain the reasoning involved in geometric proofs.
- Provide a short logical argument to support a conclusion derived from given assumptions.
- Make inferences from statistical data and explain the reasons for drawing a conclusion.

Emphasis should be placed on the use of logical arguments to communicate ideas and to convince another person of the validity of an assertion. Students should understand that a relatively small number of fundamental principles form the core of mathematical reasoning. This recognition not only facilitates clearer communication but also cultivates a deeper appreciation for the coherence and interconnectedness of mathematical concepts.

f. Functions and Their Representations

The use of functions is a major step in mathematical learning. At this higher level, an algebraic expression is treated as an entity itself—a function—and functions themselves are subject to algebraic operations. The use of algebraic operations on functions should be restricted to those that have meaning—adding, multiplying, or composing functions, translating by a shift, or rescaling of one of the variables—rather than overly complicated manipulations. Most importantly, students should appreciate the utility of functions to represent real-world phenomena and the relationships between various representations of functions.

The following are typical expectations for college-bound students:

- Create a table of values and sketch a graph from an algebraic representation of a function.
- Use function notation and the concepts of domain, co-domain, and range.
- Use graphs of functions to solve equations and inequalities.
- Apply functions as models for data while using appropriate units of measurement for inputs and outputs.
- Interpret the appropriateness of a function to model natural phenomena.

- Compute with sums, products, transformations, and compositions of functions and interpret the meaning of these operations.
- Compute the inverse of linear functions and of functions that are compositions of a linear function and a power or exponential function.
- Recognize the qualitative connections and differences between linear, polynomial, and exponential functions and the phenomena to which they apply.
- Graph, interpret, analyze, and model with essential families of functions:
  - Linear, quadratic, and integer power functions—e.g., x^{-2}, x^5—as well as simple fractional powers that are common to applications, primarily square and cube roots.
  - Absolute value, step functions, and piecewise defined functions, with applications such as pricing and tax brackets.
  - Exponential functions, with applications such as interest rates, population change, and radioactive decay.
  - Rational expressions of polynomials with a denominator of degree one, with applications such as rate problems, average profit, and elasticity of demand.

Emphasis should be placed on essential families of functions and the ways in which they appear in real-world phenomena. Understanding the qualitative similarities and differences of these functions is essential to building appropriate mathematical models. Interpreting the solutions of such models in a variety of contexts is a foundation for building a deeper understanding of the role of mathematics in the world.

5 SENIOR YEAR HIGH SCHOOL MATHEMATICS RECOMMENDATIONS BY META-MAJOR

Sections 3 and 4 of this document describe the mathematical competencies expected of all entering college students—the constellation of mathematical skills, practices, and dispositions that afford access to a wide range of quantitatively rich disciplines. This final section groups related majors into meta-majors[1] and demonstrates how the mathematical content required for college readiness is manifested within each meta-major. The descriptions of relevant mathematical content were informed by responses to a questionnaire distributed to college faculty from all three higher education segments by the subcommittee that authored this document. Suggestions for fourth-year math courses tailored to each meta-major's needs are given. References to College Board Advanced Placement (AP) exams in several fields, as well as specific problems that have mathematical content, are provided to illustrate the use of mathematical competencies that may be expected in a first-year college course.

1 A meta-major is a collection of academic programs or majors grouped together based on shared or related coursework, career paths, or fields of study. Meta-majors are designed to help students explore broad areas of interest before committing to a specific major while still making progress toward their degrees.

The objective is to equip students with the mathematical skills and dispositions necessary for success in their chosen fields of study and to prepare students for college level mathematics in the UC, CSU, and California Community Colleges systems. These recommendations assume that students possess a strong grasp of the mathematical competencies described in Sections 3 and 4. Students still honing those competencies should use the senior year to reinforce them, ideally in a course that enriches and applies the student's prior mathematical knowledge.

A fourth year of mathematics is critical for all college-bound students. This extended exposure to quantitative reasoning strengthens and sustains knowledge acquired in the first three years of high school. It fosters critical thinking, continues students' engagement with real-world problem-solving, and lays the groundwork for meeting mathematics and science general education prerequisites in college. The input from several non-STEM faculty that replied to the questionnaire can be summarized as follows: Mathematics is about more than just its direct application, and concentrating only on practical aspects risks not fully equipping students with the ability to address new and unique problems as future problem solvers.

These meta-major recommendations presuppose that students have committed to a major or have narrowed down their fields of interest. Students uncertain about their paths should consider the recommended fourth-year mathematics coursework for each potential major. Students need to understand that fourth-year mathematics course choices can both limit and expand their readiness for various major fields.

Some high schools are limited in their ability to offer a variety of fourth-year mathematics courses. The courses developed by CSU or UC programs, such as the CSU Mathematics Bridge Courses and UC Scout, can provide opportunities for students at under-resourced schools. Students may also consider taking courses at a local community college, although the availability of precalculus, trigonometry, and algebra courses at community colleges is more limited now due to AB 1705 and the guidance for implementing it provided by the California Community Colleges Chancellor's office.

School administrators and teachers should be aware of the UC Board of Admissions and Relations with Schools (BOARS) Area C Workgroup (ACW) report that distinguishes fourth-year mathematics courses into those that "build substantially on the content of the foundational sequence" of the first three years and those that do not. The following are the characteristics given in the ACW report for a data science course that builds substantially on the foundational courses (BOARS, 2024):

- The course should make substantive use of the mathematics learned in the lower-level sequence, especially in the third year.
- The algebra involved should be elegant and illuminating, not heavy-handed or labor-intensive.
- The result for the student should be a deeper understanding of mathematics as well as data science.

A course that satisfies these criteria in the area of data science is called a "mathematically rich preview of data science" in the descriptions below. These criteria are equally relevant to courses in other subjects that may be developed in the future.

a. Arts and Humanities

While students majoring in the arts and humanities may not encounter advanced mathematical concepts in the same ways as their counterparts in STEM fields, a fundamental understanding of mathematics remains important. Basic arithmetic and geometric and algebraic skills are the vocabulary and grammar that play a role in many practical tasks in the arts and in life. The feedback from arts and humanities faculty mentioned examples like the geometry of theatrical set construction, geometry in photographic techniques, and understanding the ratios involved in combining materials, such as adding water to paint or mixing plaster. These faculty also mentioned that the thorough use of logical explanations in solving mathematical problems equips students with the analytical skills needed to tackle non-quantitative questions, such as evaluating the importance of historical evidence and the reasoning behind historical interpretations.

In fact, statistical literacy, mathematical modeling, and critical analysis are increasingly valuable tools for artists and professionals in humanities-related fields as they engage with and critique culture, history, and politics. These skills are also important in general education science and math courses. While the emphasis in these courses may not be on complex mathematical theories, a foundational grasp of mathematics equips students in the arts and humanities with practical skills that enhance their ability to think critically, approach problems systematically, and engage with a diverse range of disciplines.

- Recommended fourth year of quantitative reasoning for students interested in arts and humanities: statistics, data science, discrete mathematics, or any UC approved Area C or quantitatively intense Area G class that allows the students to recognize the power of mathematics and demonstrate its relevance to their interests, majors, and career goals.
- Resources:
    - AP Human Geography Page 14: https://apcentral.collegeboard.org/media/pdf/ap-human-geography-course-and-exam-description.pdf
    - Sample AP Human Geography Problems: https://apcentral.collegeboard.org/courses/ap-human-geography/exam
    - AP World History Exam Content Page 14: https://apcentral.collegeboard.org/media/pdf/ap-world-history-modern-course-and-exam-description.pdf
    - Sample problems from AP World History: https://apcentral.collegeboard.org/courses/ap-world-history/exam/past-exam-questions

b. Behavioral and Social Sciences and Multidisciplinary Studies

Students majoring in sociology, psychology, political science, and related fields need a foundational understanding of mathematics to enhance their analytical and research skills. Proficiency in basic arithmetic, including the manipulation of fractions and percentages and the interpretation of data, is essential for conducting simple statistical analyses and interpreting research findings. Algebraic skills—solving equations and manipulating basic functions—contribute to the ability to model and analyze complex relationships within sociological and psychological frameworks. Familiarity with probability and statistics is indispensable for conducting empirical research and drawing meaningful conclusions from data. The application of geometry arises in spatial analyses or when dealing with geometric representations of data.

While advanced abstract mathematical concepts or calculus may not be necessary for students in social sciences, a solid foundation in fundamental mathematical principles empowers students to engage critically with research literature, to design and conduct experiments, and to interpret data-driven insights in their respective fields. Several faculty members from social and behavioral sciences indicated that today's compelling debates often employ quantitative components to their arguments. Examples include average temperature gain over time, carbon emissions over time, the number of weapons transferred to certain parties, or the impact of income on social, political, or academic prospects. These examples stress not only the need for mathematical skills but the need for students to practice them in context.

- Recommended fourth year course in quantitative reasoning for students interested in behavioral and social sciences: statistics, mathematically rich preview of data science, or precalculus/calculus. Students interested in quantitative social or behavioral science research should continue on the calculus pathway available to them, as these courses are the foundations for the study of more complex inferential and causal statistical methods.
- Resources
  - AP Psychology Exam (page 18-21): https://apcentral.collegeboard.org/media/pdf/ap-psychology-course-and-exam-description.pdf
  - Sample AP Psychology problems: https://apcentral.collegeboard.org/courses/ap-psychology/exam/past-exam-questions

c. Business and Economics

Students majoring in business and economics need a strong grasp of mathematics to navigate the analytical challenges inherent in these fields. Faculty in this area were particularly concerned about students gaining enough experience with mathematics in a positive environment so that they gain confidence. As discussed in Section 3, such experiences will help students feel comfortable in their mathematical abilities and in communicating about mathematics with their peers. Proficiency in arithmetic is necessary for financial calculations, budgeting, and managing resources effectively. Algebraic skills come into play when modeling and solving equations that represent economic relationships, such as supply and demand functions or the time value of money.

With the increasing access to large data sets, statistical knowledge has become increasingly important for making informed business decisions and understanding economic trends. Probabilistic concepts are employed in risk assessment and decision making under uncertainty. Calculus is applied in areas such as maximizing profit, optimization, and economic modeling. Overall, a solid foundation in mathematics equips students with the quantitative tools necessary for strategic decision making, financial analysis, and a nuanced understanding of the economic forces shaping business environments.

- Recommended fourth year course in quantitative reasoning for students interested in business and economics: precalculus/calculus, statistics, or mathematically rich preview of data science. Students intending to pursue economics or finance should continue on the precalculus and calculus pathway available to them, since they will use calculus to study topics central to their college major such as maximizing profit.
- Resources
  - AP Economics Exam Coverage Page 16: https://apcentral.collegeboard.org/media/pdf/ap-macroeconomics-course-and-exam-description.pdf
  - Sample problems from AP Economics Exam: https://apcentral.collegeboard.org/media/pdf/ap22-frq-macroeconomics-set-1.pdf

d. Biological, Medical, Environmental, and Life Sciences

Students majoring in biological and medical sciences need a robust understanding of mathematics for various aspects of research, analysis, and problem solving within these fields. Science faculty in these areas shared that their disciplines place an emphasis on students taking ownership of mathematical knowledge and applying it flexibly. Science students who enter with a strong fundamental understanding of arithmetic and algebra focus better on lab work and course content.

Proficiency in arithmetic, including skills in manipulating ratios, percentages, and concentrations, is crucial for laboratory work involving measurements and data interpretation. Algebra and functions are used to model biological processes and to solve equations that describe complex biological phenomena. Statistical knowledge is required to design experiments, analyze the data, and interpret the results in biological and medical research. Concepts such as probability are essential tools in evaluating the significance of experimental findings. Geometry plays a role in understanding three-dimensional structures, such as molecular configurations or anatomical relationships. Calculus is often applied in more quantitative areas like physiology, pharmacology, and epidemiology, providing insights into dynamic processes over time. In summary, a solid mathematical foundation empowers students in biological and medical sciences to navigate the quantitative aspects of their disciplines, fostering a more comprehensive and analytical approach to understanding living organisms and medical phenomena.

- Recommended fourth year course in quantitative reasoning for students interested in biological, medical, environmental, or life sciences: precalculus/calculus or statistics. Most students should continue on the precalculus and calculus pathway available to them, since they will use calculus to study topics central to their college majors such as the rate of muscle contraction, the rate of dissolution of drugs into the bloodstream, and the growth of bacteria. Students who have completed the calculus track courses available to them should, if possible, take statistics or a mathematically rich preview of data science.
- Resources
  - AP Biology Exam Coverage: https://apcentral.collegeboard.org/courses/ap-biology
  - AP Biology Sample Problems: https://apcentral.collegeboard.org/courses/ap-biology/exam/past-exam-questions

e. Statistics and Data Sciences

Students majoring in statistics and data sciences need a comprehensive understanding of mathematics for their academic pursuits. Proficiency in arithmetic is fundamental for handling numerical data, ensuring accuracy in calculations, and laying the groundwork for statistical analyses. Algebraic skills are used when formulating and applying statistical models. Real world data is subject to random fluctuations and measurement error, so a solid grasp of probability theory is needed to understand how drawing meaningful conclusions is still possible. Feedback from statisticians indicated that students need to visualize basic nonlinear functions. While complicated rational functions may be less critical, the capability to identify key features of functions—such as limits and points of maximum and minimum—is vital for understanding and conceptualizing mathematical functions as applied to statistics and data science.

Calculus is used broadly in these disciplines to develop optimization techniques, assess the accuracy of estimations, understand sampling techniques, and for many other statistical methods. It also undergirds advanced probability theory, which is essential for dealing with uncertainty and randomness in data. Furthermore, advanced mathematical concepts from linear algebra and computer science form the theoretical underpinnings of all data science and statistical methodologies.

- Recommended fourth year course in quantitative reasoning for students interested in statistics and data sciences: precalculus. Students in these majors will be expected to complete single variable and multivariable calculus for their college majors. Students who have completed precalculus should take a course in statistics, calculus, or a mathematically rich preview of data science.
- Resources
  - AP Statistics Exam Coverage Page 16: https://apcentral.collegeboard.org/media/pdf/ap-statistics-course-and-exam-description.pdf
  - Sample problems from AP Statistics: https://apcentral.collegeboard.org/media/pdf/ap24-frq-statistics.pdf

f. Physics, Chemistry, Engineering, Mathematics, and Computer Sciences

Students majoring in physics, chemistry, mathematics, engineering, or computer science need a profound understanding of mathematics to be successful in their academic endeavors. Arithmetic forms the basis for precise measurements, computations, and foundational numerical work in all of these disciplines. Algebra is essential for modeling physical phenomena and solving equations to get answers to physical problems. Geometry and trigonometry are used in computer graphics, when analyzing forces and motion, and in modeling and interpreting periodic behavior. Calculus is necessary for understanding the relationship between electricity and magnetism, the behavior of light and of water waves, and the energy in chemical processes. Probability and statistics play a significant role in analyzing experimental data, evaluating uncertainties, and making informed decisions in all of these fields. Logical reasoning is the basis for computer programming, troubleshooting engineering failures, and designing experiments to test a hypothesis.

As students advance in these majors, they learn linear algebra, which builds upon high school geometry and algebra, to solve systems of equations and handle multivariate data. For students in mathematics, computer science, and many fields of engineering, discrete mathematics is an essential college course that provides more training in formal logic and the relationship to circuit design, algorithm development, and optimization. A fundamental synergy exists between mathematics and these disciplines: mathematics provides the analytical tools necessary for properly defining, describing, and solving real-world problems and advancing the theoretical frameworks within physics, chemistry, mathematics, engineering, and computer science.

- Recommended fourth year of quantitative reasoning for students interested in physics, chemistry, engineering, mathematics, or computer science: precalculus/calculus. Students who have completed the calculus track courses available to them should, if possible, take statistics or more advanced mathematics courses such as multivariable calculus, differential equations, linear algebra, or discrete mathematics that may be offered at a local community college.
- Resources
  - CMF Appendix A: Key Mathematical Ideas to Promote Student Success in Introductory University Courses in Quantitative Fields: https://www.cde.ca.gov/ci/ma/cf/documents/mathfwappendixa.docx
  - Sample Problems from AP Calculus: https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions
  - AP Precalculus Exam Coverage Pages 18-19: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-course-and-exam-description.pdf
  - Sample problems from AP Precalculus: https://apcentral.collegeboard.org/media/pdf/ap-precalculus-practice-exam-multiple-choice-section.pdf

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