

APPENDIX B

SUMMARIES OF SUBJECT MATTER TOPICS WITH RELATED CALIFORNIA AND NCTM STANDARDS

This appendix lists the summaries of the subject matter topics presented in Section 2 of the Statement. After each summary, citations of related California Standards (from the *California Common Core State Standards for Mathematics*, adopted by the California State Board of Education August 2010¹) and the NCTM standards (from *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, 2000²) are given. There are two reasons for including these citations. One is to show the relationship between the Expected Competencies and the state and national standards. The second is to help teachers and other readers of the Expected Competencies find fuller descriptions of them.

The citations of the California Common Core State Standards include abbreviations of grade level or high school strand followed by the relevant subsection numbers.

The citations of the NCTM standards are grade-band specific expectations of content standards as they appear in the Appendix on pages 392-401 of *Principles and Standards*. In order to save space in this document, the standards are specified by their content area and a brief description consisting of some of their keywords.

PART 1

Essential areas of focus for all entering college students.

Variables, Equations, and Algebraic Expressions

Algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols; solutions of linear equations and inequalities; absolute value; powers and roots; solutions of quadratic equations; solving two linear equations in two unknowns including the graphical interpretation of a simultaneous solution. Emphasis should be placed on algebra both as a language for describing mathematical relationships and as a means for solving problems; algebra should not merely be the implementation of a set of rules for manipulating symbols.

CA Common Core State Standards

7.EE: Use properties of operations to generate equivalent expressions. #1, #2

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7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. #3, #4

8.EE: Work with radicals and integer exponents. #1, #2, #3, #4

8.EE: Understand the connections between proportional relationships, lines, and linear equations. #5, #6

8.EE: Analyze and solve linear equations and pairs of simultaneous linear equations. #7, #8

A-SSE: Interpret the structure of expressions. #1, #2

A-SSE: Write expressions in equivalent forms to solve problems. #3

A-APR: Perform arithmetic operations on polynomials. #1

A-APR: Understand the relationship between zeros and factors of polynomials. #2

A-CED: Create equations that describe numbers or relationships. #1

A-REI: Understand solving equations as a process of reasoning and explain the reasoning. #1, #2

A-REI: Solve equations and inequalities in one variable. #3, #4

A-REI: Solve systems of equations. #6

A-REI: Represent and solve equations and inequalities graphically. #10, #12

NCTM Standards

AL: Patterns: 9-12: generalize patterns using explicitly defined and recursively defined functions

AL: Patterns: 6-8: represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

AL: Symbols: 9-12: Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations

AL: Symbols: 9-12: Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases

AL: Symbols: 9-12: Use symbolic algebra to represent and explain mathematical relationships

AL: Symbols: 9-12: judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

AL: Symbols: 6-8: recognize and generate equivalent forms for simple algebraic expressions and solve linear equations

Families of Functions and Their Graphs

Applications; linear functions; quadratic and power functions; exponential functions; roots; operations on functions and the corresponding effects on their graphs; interpretation of graphs; function notation; functions in context, as models for data. Emphasis should be placed on various representations of functions—using graphs, tables, variables, and words—and on the interplay among the graphical and other representations; repeated manipulations of algebraic expressions should be minimized.

CA Common Core State Standards

8.F Define, evaluate, and compare functions. #1, #2, #3

8.F Use functions to model relationships between quantities. #4, #5

F-IF: Understand the concept of a function and use function notation. #1, #2

F-IF: Interpret functions that arise in applications in terms of the context. #4, #5

F-IF: Analyze functions using different representations. #7a,b #8a,b #9

F-BF: Build a function that models a relationship between two quantities. #1a

F-BF: Build new functions from existing functions. #3

F-LE: Construct and compare linear, quadratic, and exponential models and solve problems. #1, #2, #3

F-LE: Interpret expressions for functions in terms of the situation they model. #5

NCTM Standards

NO: Understand operations: 9-12: judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities

AL: Patterns: 9-12: understand relations and functions and select, convert flexibly among, and use various representations for them

AL: Patterns: 9-12: analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior

AL: Patterns: 9-12: understand and compare the properties of classes of functions, including exponential, polynomial, functions

AL: Patterns: 6-8: identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations

AL: Relationships: 9-12: identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Geometric Concepts

Distances, areas, and volumes, and their relationship with dimension; angle measurement; similarity; congruence; lines, triangles, circles, and their properties; symmetry; Pythagorean Theorem; coordinate geometry in the plane, including distance between points, midpoint, equation of a circle; introduction to coordinate geometry in three dimensions. Emphasis should be placed on developing an understanding of geometric concepts sufficient to solve unfamiliar problems and an understanding of the need for compelling geometric arguments; mere memorization of terminology and formulas should receive as little attention as possible.

CA Common Core State Standards

8.G Understand congruence and similarity using physical models, transparencies, or geometry software. #1, #2, #3, #4, #5

8.G Understand and apply the Pythagorean Theorem. #6, #7, #8

8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. #9

G-CO: Experiment with transformations in the plane. #1, #2, #3, #4, #5

G-CO: Understand congruence in terms of rigid motions. #6, #7, #8

G-CO: Prove geometric theorems. #9, #10, #11

G-CO: Make geometric constructions. #12, #13

G-SRT: Apply trigonometry to general triangles. #9, #10, #11

G-C: Understand and apply theorems about circles. #1, #2, #3, #4

G-C: Find arc lengths and areas of sectors of circles. #5

G-GPE: Use coordinates to prove simple geometric theorems algebraically. #4, #5, #6

G-GMD Explain Volume formulas and use them to solve problems. #3

NCTM Standards

GM: Synthetic: 9-12: Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them

GM: Synthetic: 6-8: Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects

GM: Analytic: 9-12: investigate conjectures and solve problems involving two- and three-dimensional objects

represented with Cartesian coordinates

GM: Transformations: 6-8: examine the congruence, similarity, and line or rotational symmetry of objects using transformations

MS: Systems: 6-8: understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume

MS: Tools: 9-12: understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Probability

Counting (permutations and combinations, multiplication principle); sample spaces; expected value; conditional probability; independence; area representations of probability. Emphasis should be placed on a conceptual understanding of discrete probability; aspects of probability that involve memorization and rote application of formulas should be minimized.

CA Common Core State Standards

7-SP Investigate chance processes and develop, use, and evaluate probability models. #5, #6, #7, #8

S-CP: Understand independence and conditional probability and use them to interpret data. #1, #2, #3, #4, #5

S-CP: Use the rules of probability to compute probabilities of compound events in a uniform probability model. #6, #7

S-MD Calculate expected values and use them to solve problems. #1, #2

NCTM Standards

NO: Understand operations: 9-12: develop an understanding of permutations and combinations as counting techniques

DA: Probability: 9-12: understand the concepts of sample space and construct sample spaces in simple cases

DA: Probability: 9-12: compute and interpret the expected value of random variables in simple cases

DA: Probability: 9-12: understand the concepts of conditional probability and independent events

DA: Probability: 6-8: compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models

Data Analysis and Statistics

Data Analysis and Statistics: Presentation and analysis of data; measures of center such as mean and

median, and measures of spread such as standard deviation and interquartile range; representative samples; using lines to fit data and make predictions. Emphasis should be placed on organizing and describing data, interpreting summaries of data, and making predictions based on the data, with common sense as a guide; algorithms should be learned with an understanding of the underlying ideas.

CA Common Core State Standards

8.SP Investigate patterns of association in bivariate data. #1, #2, #3, #4

S-ID: Summarize, represent, and interpret data on a single count or measurement variable. #1, #2, #3

S-ID: Summarize, represent, and interpret data on two categorical and quantitative variables. #6

S-ID: Interpret linear models. #7, #8, #9

S-IC: Understand and evaluate random processes underlying statistical experiments. #1, #2

NCTM Standards

DA: Data: 9-12: understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable

DA: Data: 9-12: understand histograms, parallel box plots, and scatterplots and use them to display data

DA: Statistics: 9-12: identify trends in bivariate data and find functions that model the data

DA: Statistics: 6-8: find, use, and interpret measures of center and spread, including mean and interquartile range

DA: Inferences: 6-8: make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit

Argumentation and Proof

Logical implication; hypotheses and conclusions; inductive and deductive reasoning. Emphasis should be placed on constructing and recognizing valid mathematical arguments; mathematical proofs should not be considered primarily as formal exercises.

CA Common Core State Standards

Standards for Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.

NCTM Standards

GM: Synthetic: 9-12: establish the validity of geometric conjectures using deduction, prove theorems, and

critique arguments made by others

GM: Synthetic: 6-8: create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship

PART 2

Desirable areas of focus for all entering college students (in addition to those in Part 1).

Discrete Mathematics

Topics such as set theory, graph theory, coding theory, voting systems, game theory, and decision theory.

CA Common Core State Standards

NCTM Standards

GM: Modeling: 9-12: use vertex-edge graphs to model and solve problems

Sequences and Series

Geometric and arithmetic sequences and series; the Fibonacci sequence; recursion relations.

CA Common Core State Standards

A-SSE: Write expressions in equivalent forms to solve problems. #4

F-IF. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. #3F-BF: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. #2

NCTM Standards

Geometry

Geometry: Right triangle trigonometry; transformational geometry including dilations; tessellations; solid geometry; three-dimensional coordinate geometry, including lines and planes.

CA Common Core State Standards

G-SRT: Understand similarity in terms of similarity transformations. #1, #2, #3

G-SRT: Prove theorems involving similarity. #4, #5

G-SRT: Define trigonometric ratios and solve problems involving right triangles. #6, #7, #8

G-GPE: Translate between the geometric description and the equation for a conic section. #1

G-GMD: Explain volume formulas and use them to solve problems. #3

G-GMD: Visualize relationships between two-dimensional and three-dimensional objects. #4

NCTM Standards

GM: Synthetic: 9-12: Use trigonometric relationships to determine lengths and angle measures

GM: Transformations: 9-12: understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, function notation

GM: Transformations: 9-12: use various representations to help understand the effects of simple transformations and their compositions

Number Theory

Prime numbers; prime factorization; rational and irrational numbers; triangular numbers; Pascal's triangle; Pythagorean triples.

CA Common Core State Standards

8.NS Know that there are numbers that are not rational, and approximate them by rational numbers. #1, #2

N-RN: Use properties of rational and irrational numbers. #3

A-APR Use Polynomial Identities to Solve Problems. #4

NCTM Standards

NO: Understand numbers: 9-12: compare and contrast the properties of numbers and number systems, including the rational and real numbers;

NO: Understand numbers: 9-12: use number-theory arguments to justify relationships involving whole numbers

NO: Understand numbers: 6-8: use factors, multiples, prime factorization, and relatively prime numbers to solve problems

PART 3

Essential areas of focus for students in quantitative majors (in addition to those in Parts 1 and 2)

Variables, Equations, and Algebraic Expressions

Solutions to systems of equations, and their geometrical interpretation; solutions to quadratic equations, both algebraic and graphical; complex numbers and their arithmetic; the correspondence between roots and factors of polynomials; rational expressions; the binomial theorem.

CA Common Core State Standards

N-CN: Perform arithmetic operations with complex numbers. #1, #2, #3

N-CN Represent complex numbers and their operations on the complex plane #4, #5

N-CN: Use complex numbers in polynomial identities and equations. #7 #8, #9

A-SSE Interpret the structure of expressions. #2

A-APR: Understand the relationship between the zeros and factors of polynomials. #2, #3

A-APR: Use polynomial identities to solve problems. #4, #5

A-APR: Rewrite rational expressions. #6, #7

A-CED: Create equations that describe numbers or relationships. #2, #3, #4

A-REI: Solve systems of equations. #5, #6

NCTM Standards

NO: Understand numbers: 9-12: compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions

AL: Symbols: 9-12: write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases

Functions

Rational functions; logarithmic functions, their graphs, and applications; trigonometric functions of real variables, their graphs, properties including periodicity, and applications to right triangle trigonometry; basic trigonometric identities; operations on functions, including addition, subtraction, multiplication, reciprocals, division, composition, and iteration; inverse functions and their graphs; domain and range.

CA Common Core State Standards

F-IF Analyze functions using different representations. #7c,d,e

F-BF Build a function that models a relationship between two quantities #1b,c

F-BF Build new functions from existing functions. #4, #5

F-LE Construct and Compare linear, quadratic and exponential models and solve problems. #2, #3, #4

F-TF: Extend the domain of trigonometric functions using the unit circle. #1, #2, #3, #4

F-TF: Model periodic phenomena with trigonometric functions. #5, #6, #7

F-TF: Prove and apply trigonometric identities. #8, #9

NCTM Standards

AL: Patterns: 9-12: understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions,

AL: Patterns: 9-12: understand and compare the properties of classes of functions, including rational, logarithmic, and periodic functions

Geometric Concepts

Two- and three-dimensional coordinate geometry; locus problems; polar coordinates; vectors; parametric representations of curves.

CA Common Core State Standards

N-VM Represent and model with vector quantities. #1, #2, #3

N-VM Perform operations on vectors. #4, #5

G-GPE Translate between the geometric description and the equation for a conic section. #2, #3

G-GPE Use coordinates to prove simple geometric theorems algebraically. #6, #7

G-GMD Explain volume formulas and use them to solve problems. #1, #2

G-MG: Apply geometric concepts in modeling situations. #1, #2, #3

NCTM Standards

AL: Symbols: 9-12: use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;

GM: Analytic: 9-12: use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations

Argumentation and Proof

Mathematical implication; mathematical induction and formal proof. Attention should be paid to the distinction between plausible or informal reasoning and complete or rigorous demonstrations.

CA Common Core State Standards

Standards for Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.

NCTM Standards

PART 4

Desirable areas of focus for students in quantitative majors (in addition to those in Part 3)

Vectors and Matrices

Vectors in the plane; vectors in space; dot and cross product; matrix operations and applications.

CA Common Core State Standards

N-VM: Perform operations on matrices and use matrices in applications. #6, #7, #8, #9, #10, #11, #12

A-REI Solve Systems of equations. #7, #8, #9

NCTM Standards

NO: Understand operations: 9-12: develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices

NO: Compute and estimate: 9-12: develop fluency in operations with vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more complicated cases.

Probability and Statistics

Probability and Statistics: Distributions as models; discrete distributions, such as the Binomial Distribution; continuous distributions, such as the Normal Distribution; fitting data with curves; correlation, regression; sampling, graphical displays of data.

CA Common Core State Standards

S-ID: Summarize, represent, and interpret data on two categorical and quantitative variables. #4, #5, #6

S-IC: Understand and evaluate random processes underlying statistical experiments. #1, #2

S-IC: Make inferences and justify conclusions from sample surveys, experiments, and observational studies #3, #4, #5, #6

NCTM Standards

DA: Data: 9-12: know the characteristics of well-designed studies, including the role of randomization in surveys and experiments

DA: Statistics: 9-12: for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics

DA: Statistics: 9-12: for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools

Conic Sections

Representations as plane sections of a cone; focus-directrix properties; reflective properties.

CA Common Core State Standards

G-GPE: Translate between the geometric description and the equation for a conic section. #2, #3

NCTM Standards

Non-Euclidean Geometry

History of the attempts to prove Euclid's parallel postulate; equivalent forms of the parallel postulate; models in a circle or sphere; seven-point geometry.

CA Common Core State Standards

NCTM Standards

Calculus

Calculus: A high school calculus course should have the same depth, rigor and content as university calculus courses designed for physical sciences and engineering majors. Prior to taking the course, students should have successfully completed four years of secondary school mathematics. Students completing the course should take one of the College Board's Advanced Placement Calculus examinations.

CA Common Core State Standards

NCTM Standards